Sample Paper 1

Section A

1. Select the correct answer:

(i) Let R be relation in N given by $R = \{(a,b)\,;\ a = b-2, b > 6\}$ then choose the correct answer:

(a) (2,4)∈R (b) (3,8)∈R (c) (6,8)∈R (d) (8,7)∈R

(ii) Let $f, g: R \longrightarrow R$ given by $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$ then $fog(x) =$

(a) $2x$ (b) $2x^3$ (c) $8x$ (d) none

(iii) If A $= [a_{ij}]_{m\times n}$ is a square matrix of then:

(a) $m < n$ (b) $m > n$ (c) $m = n$ (d) none

(iv) If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then $x =$

(a) 6 (b) ±6 (c) -6 (d) 6,6

(v) Find k if $f(x) = \begin{cases} kx^2, x \le 2 \\ 3, \quad x > 2 \end{cases}$ is continuous at $x = 2$.

(a) 3 (b) $\frac{4}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

(vi) $\frac{d}{dx}(\sin x^2) =$:

(a) $\cos x^2$ (b) $2x\cos x^2$ (c) $2\cos x^2$ (d) $2x\sin x^2$

(vii) The function $f(x) = 3 - 2x$ is

(a) decreasing (b) increasing (c) neither decreasing nor increasing (d) none

(viii) $\int e^{3x}dx =$

(a) $\frac{e^{3x}}{3} + c$ (b) $3e^{3x} + c$ (c) $e^{3x} + c$ (d) $3xe^{3x} + c$

(ix) $\int_{-1}^{1} x^5 \sin^3 x\, dx$

(a) 0 (b) 1 (c) 2 (d) -1

(x) Order and Degree of differential equation $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = y\frac{dy}{dx}$

(a) 1,2 (b) 2,1 (c)1,1 (d) 2,2

(xi) Find magnitude of $2\hat{\imath} + 3\hat{\jmath} + \hat{k}$

(a) $\sqrt{6}$ (b) 14 (c)$\sqrt{14}$ (d) 6

(xii) If $|\vec{a}| = 1, |\vec{b}| = 2$ and $\vec{a}.\vec{b} = 1$ then angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) 0 (d) $\frac{\pi}{3}$

(xiii) If $< l, m, n >$ are direction cosines of a vector then $l^2 + m^2 + n^2 =$

(a) 0 (b) 1 (c)3 (d) 2

(xiv) Region represented by inequalities $x \ge 0, y \ge 0$ is

(a) I quadrant (b) II quadrant (c) III quadrant (d) IV quadrant

(xv) If P(A) $= \frac{7}{13}$, P(B) $= \frac{9}{13}$ and P(A ∩ B) $= \frac{4}{13}$, find P$\left(\frac{A}{B}\right)$

(a) $\frac{4}{9}$ (b) $\frac{4}{13}$ (c) $\frac{5}{13}$ (d) $\frac{7}{9}$

2. Fill in the blanks:

(i) If $\sin^{-1} x + \cos^{-1}\left(\frac{2}{3}\right) = 1$ then $x =$ _____

(ii) If A is of 3×4 order matrix and B is of 4×2, then order of AB is _____

(iii) $\int \frac{dx}{3x-2}$=_____

(iv) The equation of a line passes through (1,0,2) and parallel to vector $3\hat{\imath} - \hat{\jmath} + 4\hat{k}$ is ___

(v) If A and B are independent events, $P(A) = a, P(B) = b$ then $P\left(\frac{B}{A}\right) =$ _____

Section B

3. Construct a 2×2 matrix whose elements are given by $a_{ij} = 2i - j$.

4. Check whether $f(x) = \begin{cases} \frac{\sqrt{1-x}-\sqrt{1+x}}{x}, & x < 0 \\ \frac{2x+1}{x-1}, & x \geq 0 \end{cases}$ is continuous at $x = 0$.

5. Find the interval in which $f(x) = x^2 + 9x - 16$ is strictly increasing or decreasing.

OR The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when $r = 10$cm.

6. Evaluate $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}\, dx$ **or** Evaluate $\int_0^1 \frac{x\,dx}{1+x^2}$

7. Find the area enclosed by $x^2 + y^2 = 9$

8. Solve $\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} = 0$

9. Find the direction cosines of a line which makes equal angles with coordinate axes.

OR If $\vec{a} = 4\hat{\imath} + 3\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{k}$ then find $|2\vec{b} \times \vec{a}|$

Section C

10. Show that the relation R in **Z** of integers is given by $R = \{(a, b); 2 \text{ divides } a - b\}$ equivalence relation.

11. If X and Y are 2×2 matrices, then solve the following equations for X and Y :

$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$, $3X + 2Y = \begin{bmatrix} 2 & 2 \\ 1 & 5 \end{bmatrix}$

12. Find $\frac{dy}{dx}$ if $x^x + y^x = 1$. ***or***

Find $\frac{dy}{dx}$, when $(i)\ x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$.

13. Evaluate $\int \frac{x+2}{2x^2+6x+5}\, dx$ **or** Evaluate $\int_{-1}^{2} |x^3 - x| dx$.

14. Solve $(x - y)dy - (x + y)dx = 0$

15. Maximize $Z = 3x + 2y$, subject to the constraints $x + 2y \leq 10, 3x + y \leq 15, x \geq 0, y \geq 0$

16. A black and a red dice are rolled. Find the conditional probability of obtaining a sum greater than 9 given that black red resulted in a 5. Or The probability of A hitting the target is $\frac{4}{5}$ and of B hitting the target is $\frac{2}{3}$ They both fire at the target. Find the probability that

(i) atleast one of them will hit the target.

(ii) Only one hit the target.

Section D

17. Solve the following by Matrix Method : $2x - 3y + 4z = 4; 3x + y - 2z = 9; 2x + 3y - 5z = 7$

18. Show that of all rectangles with given perimeter, the square has the largest area **or**

Find maximum and minimum values of $f(x) = 4x^3 - 12x^2 - 36x + 15$

19. Find the foot of perpendicular drawn from the point (2,4,3) on the line joining points $A(1,2,4)$ and B(3,4,5).