Sample Paper 2

Section A

1. Select the correct answer :

(i) If a set A has 3 elements and R be any relation in A × A then R has elements:
(a) 8 (b) 64 (c) 32 (d) 512

(ii) Let $f: R \longrightarrow R$ given by $f(x) = x^2$ then $f^{-1}(16) =$
(a) $\{-2,2\}$ (b) $\{-4,4\}$ (c) $\{-16,16\}$ (d) $\{0,4\}$

(iii) The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is:
(a) 27 (b) 18 (c) 81 (d) 512

(iv) Let A be square matrix of order 3 then $|3A| =$
(a) $27|A|$ (b) $3|A|$ (c) $9|A|$ (d) $81|A|$

(v) Find k if $f(x) = \begin{Bmatrix} kx^2, x \leq 1 \\ 4, \quad x > 1 \end{Bmatrix}$ is continuous at $x = 1$.
(a) 4 (b) $\frac{1}{4}$ (c) 1 (d) 2

(vi) $\frac{d}{dx}(\tan(2x+3)) =$
(a) $\sec^2(2x+3)$ (b) $2\tan(2x+3)$
(c) $2\sec^2(2x+3)$ (d) $2x\sec^2(2x+3)$

(vii) The function $f(x) = x^2 - 2x$ is increasing in
(a) $(-2,-1)$ (b) $(-1,0)$ (c) $(0,1)$ (d) $(1,2)$

(viii) $\int \sec x(\sec x + \tan x)dx$
(a) $\sec^2 x + \tan^2 x + c$ (b) $\sec x + \tan^2 x + c$
(c) $\sec^2 x + tanx + c$ (d) $\sec x + \tan x + c$

(ix) Find a if $\int_0^a 4x^3 dx = 16$
(a) 3 (b) 2 (c) 4 (d) 1

(x) Degree of differential equation $y''' + y^2 + e^{y'} = 0$ is
(a)3 (b) 1 (c) 2 (d) does not exist

(xi) Vector joining points P(2,3,0) and Q$(-1,-2,-4)$is:
(a) $-3\hat{\imath} - 5\hat{\jmath} - 4\hat{k}$ (b)$-3\hat{\imath} - 5\hat{\jmath} + 4\hat{k}$
(c)$-3\hat{\imath} + 5\hat{\jmath} - 4\hat{k}$ (d) $3\hat{\imath} - 5\hat{\jmath} - 4\hat{k}$

(xii) If $|\vec{a}.\vec{b}| = \sqrt{3}|\vec{a} \times \vec{b}|$ then angle between $\vec{a}$and $\vec{b}$ is
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

(xiii) Find direction cosines of the $x-$ axis are:
(a)$< 0,0,1 >$ (b)$< 1,1,1 >$ (c)$< 1,0,0 >$ (d) $< 1,1,0 >$

(xiv) Region represented by inequalities $x \geq 0, y \leq 0$ is
(a) I quadrant (b) II quadrant (c) III quadrant (d) IV quadrant

(xv) If $P(A) = 0.8, P(B) = 0.5$ and $P\left({}^{B}/_{A}\right) = 0.4$ then $P(A \cap B) =$
(a)0.4 (b) 0.32 (c) 0.8 (d) 0.5

2. Fill in the blanks:

(i) $\cos^{-1}\left(-\frac{1}{2}\right) =$ ______

(ii) If A is of 2×3 order and B is of 3×2 then order of (AB)′_______________

(iii) $\int \frac{dx}{x\log x}$=_______________

(iv) The d.r. of line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ are ____________

(v) If A and B are independent events, $P(A) = a, P(B) = b$ then $P\left(\frac{A}{B}\right) =$______

Section B

3. If A = $\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 2 & 6 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -2 \\ -4 & 5 \\ 5 & 3 \end{bmatrix}$ then find X such that 2A+3X=5B.

4. Find k if $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & x \neq \frac{\pi}{2} \\ 3, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.

5. The length x of rectangle is decreasing at the rate of 3 cm/min and width y is increasing at the rate of 2 cm/min when $x = 10$ and $y = 6$, find the rate of change of (a) the perimeter, (b) the area of rectangle. **Or**

Find the interval in which $f(x) = x^2 - 5x + 2$ is strictly decreasing.

6. If $f'(x) = x - \frac{3}{x^2}, f(1) = \frac{11}{2}$, find $f(x)$. **or** Evaluate $\int_0^{\pi/2} \frac{\sin x \, dx}{1+\cos^2 x}$

7. Find the area enclosed by $x^2 + y^2 = 4$.

8. Solve $y\log y . dx - x . dy = 0$

9. Find the vectors and Cartesian equation of the line through the $(5,2,-4)$ and parallel to vector $3\hat{\imath} + 2\hat{\jmath} - 8\hat{k}$. OR Prove that Cauchy schwartz inequality: For any two vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}.\vec{b}| \leq |\vec{a}||\vec{b}|$

Section C

10. Show that the relation R in **R** defined as $R = \{(a, b); a \leq b\}$ is reflexive and transitive but not symmetric.

11. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, verify that $A^2 - 5A + 7I = 0$

12. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$. ***or*** Differentiate $\log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ w.r.t. x

13. Evaluate $\int \frac{x+3}{\sqrt{5-4x-x^2}} dx$ **or** Evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$.

14. Solve $x^2 dy = (2xy + y^2)dx$

15. Maximize $Z = 4x + y$ subject to the constraints: $x + y \leq 50, 3x + y \leq 90, x \geq 0, y \geq 0$

16. A and B appeared for an interview. the probability of their selection is $\frac{1}{3}$ and $\frac{1}{6}$ respectively. Find the probability (i) both selected (ii) only one selected (iii) none selected. **Or**

An urn contains 10 black and 5 white balls. Two balls are drawn from urn one after other without replacement, what is the probability that both balls are black.

Section D

17. Solve $3x + 4y + 7z = 4; 2x - y + 3z = -3; x + 2y - 3z = 8$ **or**

Express $\begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$ as sum of symmetric and skew-symmetric matrices.

18. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius a is a square of side $\sqrt{2}a$. **Or** Show that a closed right circular cylinder of given total surface area and maximum volume is such that its height is equal to the diameter of its base.

19. Find the equation of the line passing through $(1,2,-4)$ and perpendicular to the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-5}{2} = \frac{y-2}{8} = \frac{z+5}{-5}$.