Sample Paper 3

Section A

1. Select the correct answer :

(i) If $f(x) = 3x$ and $g(x) = \sin x$ where $f: R \longrightarrow R$ and $g: R \longrightarrow R$ then $fog(x) =$

(a) $\sin 3x$ (b) $3\sin x$ (c) $3x\sin x$ (d) $3\sin 3x$

(ii) The relation given by $R = \{(1,1),(1,2),(2,1),(2,2)\}$ is

(a) Reflexive (b)symmetric (c) Transitive (d) Equivalence

(iii) If A and B are symmetric matrices of same order then AB − BA is

(a) skew symmetric (b) symmetric (c) zero (d) identity matrix

(iv) If $A = \begin{bmatrix} m & -2 \\ -2 & m \end{bmatrix}$ and $|A|^2 = 25$ then $m =$

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

(v) Find a if $f(x) = \begin{Bmatrix} \frac{\sin 2x}{x}, x \neq 0 \\ a, \quad x = 0 \end{Bmatrix}$ is continuous at $x = 0$.

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

(vi) $\frac{d}{dx}\left(\cos\sqrt{x}\right) =$:

$(a)\ \sin\sqrt{x}$ (b) $\frac{-\sin\sqrt{x}}{2\sqrt{x}}$ (c)$\frac{-\sin\sqrt{x}}{\sqrt{x}}$ (d) $\frac{\sin\sqrt{x}}{2\sqrt{x}}$

(vii) The function $f(x) = -x^2 + 6x - 3$ is decreasing in

(a) $x < 3$ (b) $x > -3$ (c) $x > 3$ (d) $x < -3$

(viii) $\int (5 - 3x)^3 dx =$

(a)$\frac{(5-3x)^4}{-12} + c$ (b) $\frac{(5-3x)^4}{4} + c$ (c) $-12(5 - 3x)^4 + c$ (d) $12(5 - 3x)^4 + c$

(ix) $\int_1^2 (2x + 3)\, dx =$

(a) 6 (b) 4 (c) 5 (d) 2

(x) Degree of differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) = 1$ is

(a) 3 (b) 2 (c)1 (d) does not exist

(xi) If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 80$ then find $|\vec{x}|$

(a) 81 (b) 9 (c) 1 (d) 0

(xii) If $|\vec{a}| = 1, |\vec{b}| = 2$ and $\vec{a}.\vec{b} = 1$ then angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(xiii) If the lines $\frac{x-5}{-3} = \frac{y-2}{2} = \frac{z+5}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ are perpendicular then k=

(a)−2 (b)−5 (c) 5 (d) $\frac{3}{2}$

(xiv) Solution set of inequality $y \leq 0$ is

(a) half plane below x-axis excluding points on x-axis

(b) half plane below x-axis including points on x-axis

(c)half plane above x-axis (d) none of these

(xv) If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$ find $P(A \cap B)$ if A and B are independent events:

(a) $\frac{4}{5}$ (b) $\frac{2}{5}$ (c)$\frac{3}{25}$ (d) $\frac{4}{25}$

2. Fill in the blanks:

(i) $\sin(\cos^{-1} x)$ ______

(ii) If $\begin{bmatrix} 5 & 3x \\ 2y & 4 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 6 & 4 \end{bmatrix}'$ then xy=___________

(iii) $\int \cot^2 x dx =$ ____________

(iv) If a line makes angles α, β and γ with coordinate axes then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$___________

(v) If A and B are independent events, $P(A) = a, P(B) = b$ then $P(A \cap B) =$ ______

Section B

3. Find X and Y if $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}, X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$.
4. Differentiate $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ w.r.t. x
5. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when $r = 10$cm. Or
   Find the interval in which $f(x) = \sin 2x$ is strictly increasing .
6. Evaluate $\int \frac{x}{\sqrt{x+4}} dx$ or Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$
7. Solve $e^x \tan y . dx + (1 - e^x). \sec^2 y. dy = 0$
8. Find the area enclosed by $y^2 = 4x, x = 1$ *and* $x = 3$ in first quadrant.
9. Prove that for any two vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$ or Find the angle between pair of lines given by $\vec{r} = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ and $\vec{r} = 5\hat{\imath} - 2\hat{\jmath} + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$.

Section C

10. Let L be set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1, L_2); L_1$ is parallel to $L_2\}$, show that R is an equivalence relation.
11. If $f(x) = x^2 - 2x - 3$, find f(A) where $A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$.
12. If $y = \sin^{-1}x$ then show that $(1 - x^2)y_2 - xy_1 = 0$. **OR**
    If $y = x\sin(a + y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y) - y\cos(a+y)}$
13. Evaluate $\int \frac{3x-2}{(x+1)^2(x+3)} dx$ **or** Evaluate $\int_0^{\pi} \frac{x}{1+\sin x} dx$.
14. Solve $x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$
15. Maximize $Z = 3x + 4y: x + 2y \le 8, 3x + 2y \le 12, x, y \ge 0$
16. In a factory which manufactures bolts: machine A, B and C manufactures 25%, 35% and 40% of the bolts respectively. Of their outputs 5%, 4% and 2% are respective defective bolts. A bolt is drawn at random from product and found to be defective. What is the probability that it is manufactured by the machine A? **OR** A problem is given to three students, whose chance of solving it are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{2}$ respectively. Find the probability that (i) exactly one will solve (ii) problem is solved.

Section D

17. Solve $x - y + 2z = 1, 2y - 3z = 1 , 3x - 2y + 4z = 2$
18. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and other into a circle. What should be the length of two pieces so that the combined area of the square and the circle is minimum. **Or** Show that semi-vertical angle of the right circular cone of the given slant height and maximum volume is $\tan^{-1}\sqrt{2}$.
19. Find the length of the perpendicular drawn from the point (1,2,3) on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-3}{-2}$. Also find foot of perpendicular. **Or** Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ and each one of them being perpendicular to the sum of other two then find $|\vec{a} + \vec{b} + \vec{c}|$.