Sample Paper 4

Section A

1. Select the correct answer :

(i) Let $f: R \longrightarrow R$ given by $f(x) = 3x - 4$ then $f(x)$ is

(a) One-one (b)onto (c) one-one and onto (d) none

(ii) Let $A = \{1,2,3\}$ and consider the relation $R = \{(1,1),(2,2),(3,3),(1,2)\}$ then R is

(a) Reflexive but not symmetric (b)reflexive but not transitive

(c) symmetric and transitive (d) neither symmetric nor transitive

(iii) If matrix A is both symmetric and skew symmetric then A is:

(a) diagonal matrix (b) zero matrix

(c) identity matrix (d) none

(iv) Let A be non-singular matrix of order 3×3, then |Adj. A| =

(a) $|A|^3$ (b) $|A|$ (c) $|A|^2$ (d) $|A|^2$

(v) Find a if $f(x) = \begin{Bmatrix} \frac{x^2-9}{x-3}, x \neq 3 \\ a, \quad x = 3 \end{Bmatrix}$ is continuous at $x = 3$.

(a) 3 (b) 6 (c) 0 (d) 9

(vi) If $x - y = \pi$ then $\frac{dy}{dx} =$:

(a)π (b) 2 (c) 1 (d) $\pi - 1$

(vii) The function $f(x) = x^2 e^{-x}$ is increasing in

(a) $(-\infty, \infty)$ (b) $(-2,0)$ (c) $(2, \infty)$ (d) $(0,2)$

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(viii) $\int \sin 4x dx =$

(a)$4\cos 4x + c$ (b) $-4\cos 4x + c$ (c) $\frac{\cos 4x}{4} + c$ (d) $\frac{-\cos 4x}{4} + c$

(ix) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

Abhyaas:

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

(x) The number of arbitrary constants in the general solution of a differential equation of order 4 are:

(a) 3 (b) 2 (c)1 (d) 4

(xi) Find a unit vector in direction of $2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$

(a) $2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ (b) $\frac{2\hat{\imath}+3\hat{\jmath}-6\hat{k}}{7}$ (c) $-2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$ (d) $\frac{2\hat{\imath}-3\hat{\jmath}+6\hat{k}}{7}$

(xii) Find angle between $\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\widehat{3\imath} - 2\hat{\jmath} + \hat{k}$:

(a) $\cos^{-1}\frac{5}{7}$ (b) $\cos^{-1}\frac{7}{5}$ (c) $\cos^{-1}\frac{10}{7}$ (d) $\cos^{-1}\frac{5}{14}$

(xiii) The point which lies on the line $\frac{x+1}{2} = \frac{y-2}{3} = \frac{z+1}{4}$ is:

(a) $(-1,2,-1)$ (b) $(1,-2,1)$ (c)$(1,2,1)$ (d) $(-1,-2,1)$

(xiv) The point which lies in half plane of $3x - y \geq 3$ is

(a) (0,0) (b) (2,0) (c) (1,2) (d) (0,1)

(xv) If $P(A) = a$ and $P(B) = b$ are mutually exclusive events then P(A∩B)=

(a)0 (b) ab (c) $a + b$ (d) $a - b$

2. Fill in the blanks:

(i) $\sin(\cos^{-1} x + \sin^{-1} x) =$ ______

(ii) If A = $[a_{ij}]_{m\times n}$ is a square matrix if________

(iii) $\int_{-2}^{2}(\sin x + x^3)\, dx =$ ________

(iv) The d.r. of the line $\frac{2x+1}{4}=\frac{2-y}{3}=\frac{3z+5}{9}$ are __________

(v) If $P(A)=0.8, P(B)=0.5, P(A\cap B)=0.4$ then $P(AUB)=$

Section B

3. Find x and y if $2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix}+\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 6\\ 15 & 14\end{bmatrix}$.

4. Differentiate $\tan^{-1}\left(\frac{2+3x}{3-2x}\right)$ w.r.t x

5. Find the point on the curve $y^2=8x$ for which the abscissa and ordinate change at the same rate. Or Find the interval in $f(x)=x^2+9x-16$ is strictly increasing or decreasing.

6. Evaluate $\int\frac{\cos 2x}{(\cos x+\sin x)^2}\,dx$ or Evaluate $\int_0^1\frac{e^x dx}{1+e^{2x}}$

7. Find the area enclosed by $y^2=4x, x=1$ and $x=2$ and in first quadrant.

8. Solve $(1+e^{2x}).dy+(1+y^2).e^x.dx=0$

9. Find p so that the lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-2}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. Or

   Find the scalar projection of $\vec{a}=2\hat{\imath}+3\hat{\jmath}+2\hat{k}$ on $\vec{b}=\hat{\imath}+2\hat{\jmath}+\hat{k}$.

Section C

10. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$.

11. If $A=\begin{bmatrix}3 & 4\\ 4 & 5\end{bmatrix}$, $B=\begin{bmatrix}5 & 3\\ 2 & 1\end{bmatrix}$, verify $(AB)'=B'A'$.

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12. If $y=e^{a\cos^{-1}x}$ then show that $(1-x^2)y_2-xy_1-a^2y=0$. **Or**

    If $x^y=e^{x-y}$, prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$

13. Evaluate $\int\frac{(3\sin x-2)\cos x}{5-\cos^2x-4\sin x}\,dx$ **or** Evaluate $\int_0^1\frac{\log(1+x)}{1+x^2}\,dx$.

14. Solve $x.\frac{dy}{dx}-y-2x^3=0$

15. Minimize $Z=3x+5y$ subject to the constraints: $x+3y\geq 3, x+y\geq 2, x,y\geq 0$.

16. There are two bags, First bag contains 4 white and 3 red balls, 2nd bag contains 6 white and 5 red balls. One ball is drawn at random from one of the bags and found to be red. Find the probability that it is drawn from 2nd bag. OR A and B appeared for an interview. the probability of their selection is $\frac{1}{3}$ and $\frac{1}{6}$ respectively. Find the probability (i) both selected (ii) only one selected (iii) none selected.

Section D

17. Solve the following by Matrix Method: $\frac{4}{x}-\frac{3}{y}+\frac{10}{z}=2$; $\frac{8}{x}+\frac{6}{y}+\frac{5}{z}=5$; $\frac{-12}{x}+\frac{3}{y}+\frac{15}{z}=1$

    OR If $A=\begin{bmatrix}3 & -3 & 4\\ 0 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$, show that $A^4=I$. Hence find A^{-1}.

18. Show that a closed right circular cylinder of given total surface area and maximum volume is such that its height is equal to the diameter of its base. **Or** Find two positive numbers whose sum is 15 and the sum of whose squares is minimum

19. Show that the lines $\vec{r}=3\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(\hat{\imath}+2\hat{\jmath}+2\hat{k})$ and $\vec{r}=5\hat{\imath}-2\hat{\jmath}+\mu(3\hat{\imath}+2\hat{\jmath}+6\hat{k})$ are intersecting. Also find point of intersection.