Sample Paper 5

Section A

1. Select the correct answer :

(i) Let $n(A) = p$ then number of relations in $A \times A$ are

(a) 2^{2p} (b) 2^{p^2} (c) p^2 (d) 2^p

(ii) Let $f: R \longrightarrow R$ given by $f(x) = \frac{3x+1}{2}$ then $f(x)$ is

(a) One-one (b)onto (c) one-one and onto (d) none

(iii) If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then $x =$

(a) 6 (b) ± 6 (c) -6 (d) 0

(iv) If A is a $(x+2) \times (y-3)$ matrix and B is a 2×5 matrix and AB is a 3×5 then x and y.

(a) 5,1 (b) 2,3 (c) 1, 5 (d) 3,2

(v) $\lim_{x \to 0} \frac{2^{3x}-1}{x} =$

(a) log2 (b) 1 (c) 3log2 (d) 3

(vi) $\frac{d}{dx} \cos^{-1}(\sin x) = :$

(a) -1 (b) 1 (c) $\frac{1}{\sqrt{1-x^2}}$ (d) $-\cos^{-1} x$

(vii) The function $f(x) = \sin 2x + 6$ is decreasing in

(a) $(0, \pi)$ (b) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (c) $\left(0, \frac{\pi}{4}\right)$ (d) $\left(\pi, \frac{5\pi}{4}\right)$

(viii) $\int \tan x dx =$

(a) $\sec^2 x + c$ (b) $\log|\cos x| + c$ (c) $\log|\sec x| + c$ (d) $\log|\sin x| + c$

(ix) $\int_0^1 \frac{dx}{1+x^2}$

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

(x) The number of arbitrary constants in particular solution of a diff. equation of order 3 are:

(a) 3 (b) 2 (c)1 (d) 0

(xi) Find a if the vectors $\vec{x} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{y} = a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear?

(a) 4 (b) -4 (c) 2 (d) -2

(xii) If θ is the angle between $\vec{a}$ and $\vec{b}$ and $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$ then $\theta =$

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

(xiii) Any point lie on the line $\frac{x+1}{2} = \frac{y-2}{3} = \frac{z+3}{3}$ is:

(a)$(1,-2,3)$ (b)$(-1,2,3)$ (c) $(-1,2,-3)$ (d) $(1,-2,-3)$

(xiv) The point which lies in half plane of $x - y \geq 0$ is

(a) (0,0) (b) (2,0) (c) (1,2) (d) (0,1)

(xv) If $P(A) = \frac{6}{11}, P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$, find $P(A/B)$:

(a) $\frac{2}{3}$ (b) $\frac{5}{6}$ (c) $\frac{4}{5}$ (d) $\frac{3}{5}$

2. Fill in the blanks:

(i) $\cos(\sin^{-1} x) =$ ______

(ii) If $\begin{bmatrix} x+2y & 3y \\ 4x & 2 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 8 & 2 \end{bmatrix}$ then value of $x + y =$____________

(iii) If $\int_0^a 3x^2 dx = 8$ then $a =$__________

(iv) If lines $\frac{x-1}{2}=\frac{y+3}{5}=\frac{z-1}{a}$ and $\frac{x+2}{3}=\frac{y-2}{2}=\frac{z+5}{4}$ are perpendicular then $a=$ _______

(v) If A and B are mutually exclusive events, $P(A)=a, P(B)=b$ then $P(A\cap B)=$ ____

Section B

3. Find AB if $A=\begin{bmatrix}6 & 9\\2 & 3\end{bmatrix}$, $B=\begin{bmatrix}2 & 6 & 0\\7 & 9 & 8\end{bmatrix}$.
4. Find k if $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, x\neq\frac{\pi}{2}\\ 3, x=\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$.
5. Find the interval in which the function $f(x)=2x^3-3x^2-36x+7$ is strictly increasing or decreasing. **Or** An edge of a variable cube is increasing at the rate of 3 cm/sec. How fast is the volume of the cube increasing when edge is 10 cm long?
6. Evaluate $\int \sin 3x.\cos 4x\ dx$ **OR** Evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\ dx$
7. Find the area enclosed by $y^2=4x, x=2$ and $x=5$ and above $x-$ axis..
8. Solve $(1+y^2)(1+\log x).dx+x.dy=0$
9. Find the area of parallelogram whose adjacent sides are $3\hat{\imath}+\hat{\jmath}+4\hat{k}$ and $\hat{\imath}-\hat{\jmath}+\hat{k}$. **or** Find the equation of a line passing through $(2,-3,4)$ and parallel to line $\vec{r}=2\hat{\imath}-3\hat{\jmath}+4\hat{k}+\lambda(\hat{\imath}-\hat{\jmath}+2\hat{k})$

Section C

10. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x\in\left[0,\frac{\pi}{4}\right]$.

11. If $A=\begin{bmatrix}2 & -1 & 3\\1 & 2 & 1\\4 & 3 & -1\end{bmatrix}$, then show that $|2A|=8|A|$.

12. If $y=\left[\log\left[x+\sqrt{x^2+1}\right]\right]^2$ then show that $(1+x^2)y_2+xy_1=2$ **or**
If $\sin y=x\sin(a+y)$, prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$.

13. Evaluate $\int\frac{x^2+x+1}{(x+2)(x^2+1)}\ dx$

14. Solve $(1+y^2)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$

15. Maximize $Z=300x+200y$: $x+y\leq 24, 2x+y\leq 32, x,y\geq 0$

16. A problem is given to three students, whose chance of solving it are $\frac{1}{4},\frac{1}{3}$ and $\frac{1}{2}$ respectively. Find the probability that (i) exactly one will solve (ii) problem is solved. OR There are two bags, First bag contains 4 wite and 3 red balls, 2nd bag contains 6 white and 5 red balls. One ball is drawn at random from one of the bags and found to be red. Find the probability that it is drawn from 2nd bag.

Section D

17. Solve: $x+y+z=6, 2x-y+z=3, x-2y+3z=6$.

18. Show that semi-vertical angle of the right circular cone of the given slant height and maximum volume is $\tan^{-1}\sqrt{2}$. **or** An open box with square base is to be made out of a given iron sheet of area 36 m^2. Show that the maximum volume of the box is $12\sqrt{3}$ m^3.
19. Find shortest distance between lines $\vec{r}=\hat{\imath}+\hat{\jmath}+\lambda(2\hat{\imath}-\hat{\jmath}+\hat{k})$ and $\vec{r}=2\hat{\imath}+\hat{\jmath}-\hat{k}+\mu(3\hat{\imath}-5\hat{\jmath}+2\hat{k})$ **or** Express $\vec{a}=5\hat{\imath}-2\hat{\jmath}+5\hat{k}$ as sum of two vectors such that one is parallel to the vector $\vec{b}=3\hat{\imath}+\hat{k}$ and other is perpendicular to $\vec{b}$.